Moveability and Collision Analysis for Fully-Parallel Manipulators

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Abstract

The aim of this paper is to characterize the moveability of fully-parallel manipulators in the presence of obstacles. Fully parallel manipulators are used in applications where accuracy, stiffness or high speeds and accelerations are required [1]. However, one of its main drawbacks is a relatively small workspace compared to the one of serial manipulators. This is due mainly to the existence of potential internal collisions, and the existence of singularities. In this paper, the notion of free aspect is defined which permits to exhibit domains of the workspace and the joint space free of singularity and collision. The main application of this study is the moveability analysis in the workspace of the manipulator as well as path-planning, control and design.

Key Words: Parallel Manipulator, Singularity, Working Modes, Collisions, Aspect, Moveability, Design.

1 Introduction

The aim of this paper is to characterize the moveability of fully-parallel manipulators in the presence of obstacles. Fully parallel manipulators are used in applications where accuracy, stiffness or high speeds and accelerations are required [1]. However, one of its main drawbacks is a relatively small workspace compared to the one of serial manipulators. This is due mainly to the existence of potential internal collisions, i.e. collisions between different bodies of the manipulator. The most current method to avoid such collisions is to limit the range of the actuated and passive joints by constant values. This reduces the workspace more than needed. A better method is to consider virtual limits by software. By adjusting these limits in function of the configuration of the manipulator, a significant part of the workspace would be saved. Another feature which seriously

reduces the workspace of fully-parallel manipulators is the existence of singularities. It is well known that two Jacobian matrices appear in the kinematic relations between the joint-rate and the Cartesian-velocity vectors, which are called the "inverse kinematics" and the "direct kinematics" matrices. The study of these matrices allows to define the parallel and the serial singularities, respectively [2]. They appear when two solutions of the direct kinematics (resp. inverse kinematics) meet. A parallel singularity generally appears inside the workspace and is very difficult to cross.
The notion of collision-free space is introduced to take into account the internal/external collisions. However, its projection onto the workspace is insufficient to conclude as to the moveability of the manipulator. To solve this problem, we define the notion of free aspect for general fully parallel manipulators in the presence of obstacles.
This study is illustrated with a RR-RRR planar parallel manipulator.

2 Preliminaries

2.1 The fully parallel manipulators

Definition 1 *A fully parallel manipulator is a mechanism that includes as many elementary kinematic chains as the mobile platform does admit degrees of freedom. Moreover, every elementary kinematic chain possesses only one actuated joint (prismatic, pivot or kneecap). Besides, no segment of an elementary kinematic chain can be linked to more than two bodies [1].*

In this study, kinematic chains, or legs [4], are always independent.

2.2 Kinematic equations

The vector of input variables $\mathbf{q}$ and the vector of output variables $\mathbf{X}$ for a n-DOF fully parallel manipulator are related through a system of non linear algebraic equations which can be written as

$$F(\mathbf{X}, \mathbf{q}) = \mathbf{0} \tag{1}$$

where $\mathbf{0}$ means here the n-dimensional zero vector. $\mathbf{q}$ is the vector of actuated joint variables: $\mathbf{q} \in Q$, where Q is referred to as the joint space of the manipulator. $\mathbf{X}$ is the vector of position and orientation of the moving platform: $\mathbf{X} \in W$ where W is the workspace of the manipulator. The position and orientation of all the bodies of the manipulator are fully defined by ($\mathbf{X}$, $\mathbf{q}$) which will be referred to as *mechanism configuration* in this paper. Differentiating (1) with respect to time leads to the velocity model

$$\mathbf{At} + \mathbf{B}\dot{\mathbf{q}} = \mathbf{0}$$

The parallel singularities (resp. serial singularities) occur when the determinant of direct kinematic matrix $\mathbf{A}$ (resp. $\mathbf{B}$) vanishes [6]. One can remark that for fully parallel manipulators, $\mathbf{B}$ is always diagonal [6].

2.3 Working modes

The *working modes* were defined in [6] for n-DOF fully parallel manipulators as follows: A *working mode*, denoted Mf_i, is the set of mechanism configurations for which the sign of $\mathbf{B}_{jj}(j = 1 \ to \ n)$ does not change and $\mathbf{B}_{jj}$ does not vanish.

$$Mf_i = \left\{ (\mathbf{X}, \mathbf{q}) \in W \cdot Q \setminus \begin{array}{r} sign(\mathbf{B}_{jj}) = constant \ for \ (j = 1 \ to \ n) \\ and \ det(\mathbf{B}) \neq 0 \end{array} \right\}$$

where $W \cdot Q$ means the cartesian product of W by Q.
Therefore, the set of working modes ($Mf = \{Mf_i\}, i \in I$) is obtained while using all permutations of sign of each term $\mathbf{B}_{jj}$.

2.4 The manipulator and the environment

Definition 2 *Let* $\mathbf{V_M}(\mathbf{X}, \mathbf{q})$*, the volume of the fully parallel manipulator in the mechanism configuration* $(\mathbf{X}, \mathbf{q})$.

$$\mathbf{V_M}(\mathbf{X}, \mathbf{q}) = \mathbf{b} \cup (\cup_{k=0,n\times m-1}\mathbf{c}_k(\mathbf{X}, \mathbf{q})) \cup \mathbf{pl}(\mathbf{X})$$

where

- $\mathbf{b}$ is the volume of the fixed base of the fully parallel manipulator;
- $\mathbf{pl}(\mathbf{X})$ is the volume of the mobile platform of the fully parallel manipulator in the platform configuration $(\mathbf{X})$;
- $\mathbf{c}_k(\mathbf{X}, \mathbf{q})$ with $k = m \times i + j$ is the volume of the link j of the leg i where n is the number of legs and m the number of links of the leg i of the fully parallel manipulator.

Definition 3 *Let* $\mathbf{V_{ic}}(\mathbf{X}, \mathbf{q})$, the volume of the internal collisions, *i.e. the set of all the volumes in collision between the links of the manipulator in the mechanism configuration* $(\mathbf{X}, \mathbf{q})$ *(Figure 1):*

$$\mathbf{V_{ic}}(\mathbf{X}, \mathbf{q}) = \begin{array}{l} (\cup_{i=0,n\times m-1}(\mathbf{c}_i \cap \mathbf{b})) \ \cup \ (\cup_{i=0,n\times m-1}(\mathbf{c}_i \cap \mathbf{pl}(\mathbf{X}))) \ \cup \\ (\cup_{i=0,n\times m-1}(\cup_{j=i+1,n\times m-1}(\mathbf{c}_i \cap \mathbf{c}_j))) \end{array}$$

Definition 4 *Let* $\mathbf{V_{ec}}$*,* the volume of the external collisions*, i.e. the set of all the volumes in collision between the mechanism and the obstacles (Figure 2):*

$$\mathbf{V_{ec}} = \mathbf{V_M} \cap (\cup_{s=1,No} \mathbf{Obst}_s)$$

where No is the number of obstacles.

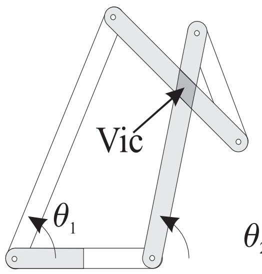

Fig. 1: Example of internal collision

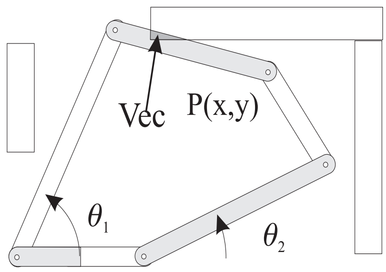

Fig. 2: Example of external collsion

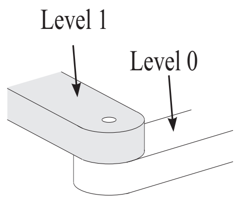

Fig. 3: Example of revolute

3 The free aspects of fully parallel manipulators

3.1 Collision-free space, workspace and joint space

Definition 5 *Let* S_{CF}*, the* collision-free space *of the fully parallel manipulator:*

$$S_{CF} = \{(\mathbf{X}, \mathbf{q}) \in W \cdot Q \setminus \mathbf{V_{ec}}(\mathbf{X}, \mathbf{q}) = \emptyset \text{ and } \mathbf{V_{ic}}(\mathbf{X}, \mathbf{q}) = \emptyset\}$$

A free mechanism configuration $(\mathbf{X}, \mathbf{q})$ is a mechanism configuration for which the manipulator is free of internal and external collisions.

Definition 6 *The projection* $\mathbf{\Pi}_W$ *of the collision-free space* S_{CF} *onto the workspace yields the* collision-free workspace W_F*:*

$$W_F = \mathbf{\Pi}_W(S_{CF})$$

Definition 7 *The projection* $\mathbf{\Pi}_Q$ *of the collision-free space* S_{CF} *onto the joint space yields the* collision-free joint space Q_F*:*

$$Q_F = \mathbf{\Pi}_Q(S_{CF})$$

3.2 The free aspect

The notion of aspect was introduced by [5] to cope with the existence of multiple inverse kinematic solutions in serial manipulators. Recently, the notion of aspect was defined for fully parallel manipulators with only one inverse kinematic solution [3] and for fully parallel manipulators with several inverse and direct kinematic solutions [6]. However, no collision was considered in these last definitions.
In this section, the notion of free aspect is defined formally for fully parallel manipulators in the presence of collisions.

Definition 8 *The free aspects A_{Fij} are defined as the maximal sets in $W \cdot Q$ so that*

- $A_{Fij} \subset W \cdot Q$;
- A_{Fij} is connected.
- $A_{Fij} = \{(\mathbf{X}, \mathbf{q}) \in Mf_i \setminus det(\mathbf{A}) \neq 0 \textit{ and } \mathbf{V_{ec}}(\mathbf{X}, \mathbf{q}) = \emptyset \textit{ and } \mathbf{V_{ic}}(\mathbf{X}, \mathbf{q}) = \emptyset\}$

In other words, the free aspects A_{Fij} are the maximal singularity-free domains of $W \cdot Q$ without neither internal nor external collisions.

Definition 9 *The projection $\mathbf{\Pi}_W$ of the free aspects onto the workspace yields the free W-aspects WA_{Fij}:*

- $WA_{Fij} = \mathbf{\Pi}_W(A_{Fi})$;
- WA_{Fij} is connected.

The free W-aspects are the maximal singularity-free domains in the workspace without neither internal nor external collisions.

Definition 10 *The projection $\mathbf{\Pi}_Q$ of the free aspects in the joint space yields the free Q-aspects QA_{Fij}:*

- $QA_{Fij} = \mathbf{\Pi}_Q(A_{Fi})$;
- QA_{Fij} is connected.

The free Q-aspects are the maximal singularity-free domains in the joint space without neither internal nor external collisions.

4 Applicative example

In this section, a planar manipulator is used as illustrative example. This is a five-bar, revolute (R)-closed-loop linkage, as displayed in figure 4. The actuated joint variables are θ_1 and θ_2, while the output variables are the (x, y) coordinates of the revolute center P. The passive and the active joints will be assumed unlimited in this study. Lengths L_0, L_1, L_2, L_3, and L_4 define completely the geometry of this manipulator. We assume here the dimensions $L_0 = 8$, $L_1 = 7$, $L_2 = 7$, $L_3 = 5$ and $L_4 = 5$, in certain units of length that we need not specify.
In this example, the environment is assumed free of obstacles so that only internal collisions are taken into account. The quadtree model is used to represent the moveability regions in the workspace and the joint space. They are calculated using discretization and enrichment techniques [7].

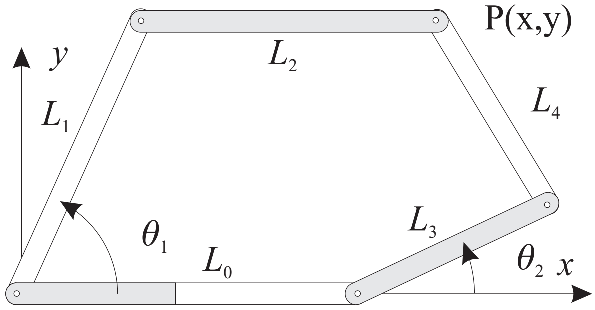

Fig. 4: Planar fully parallel manipulator

4.1 Collision Free Workspace and Joint Space

In this example, the collision free joint space (Figure 5) is smaller than the joint space (Figure 6) but the collision-free workspace is similar to the workspace (Figure 7).

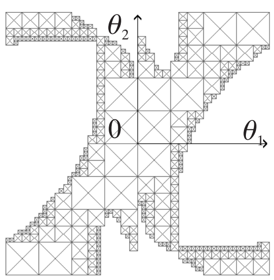

Fig. 5: Collision free joint space

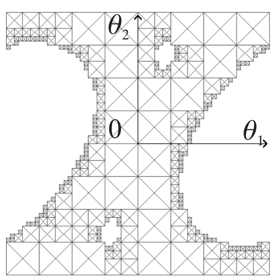

Fig. 6: Joint space

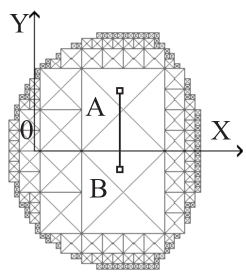

Fig. 7: Workspace and collision-free workspace

However, in this workspace, any trajectory is not possible. For example, the trajectory (AB) is infeasible because P goes through the fixed base (L_0) yielding a collision.

Therefore, the free workspace is insufficient to conclude as to the moveability of the manipulator. We need to compute the free aspects.

4.2 Free aspect

In this section, the free W-aspects and free Q-aspects are displayed in the case where $det(A) > 0$ for the 4 existing working modes (Figure 8). The other aspects ($det(A) < 0$) are located symmetrically with regard to the X axis.

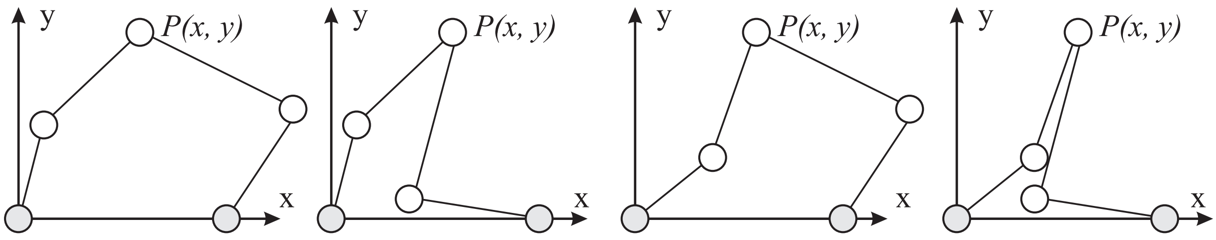

Fig. 8: The four working modes of the RR-RRR manipulator

We can remark that the number of aspects varies according to the working mode (Figure 9). The boundary of the aspects are defined by the parallel and serial singularities as well as the collisions.

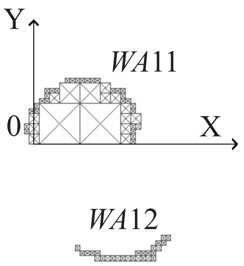

Fig. 9: Free W-aspects WA_{11} and WA_{12}

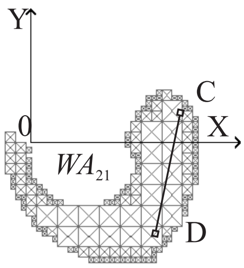

Fig. 10: Free W-aspect WA_{21}

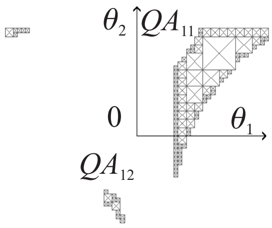

Fig. 11: Free Q-aspects QA_{11} and QA_{12}

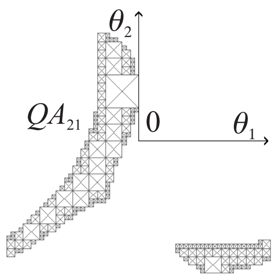

Fig. 12: Free Q-aspect QA_{21}

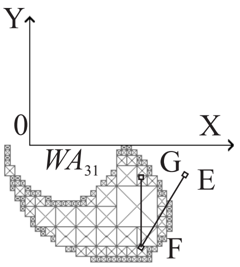

Fig. 13: Free W-aspect WA_{31}

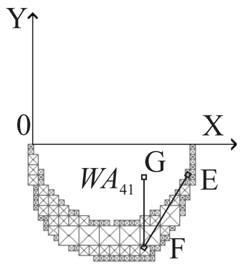

Fig. 14: Free W-aspect WA_{41}

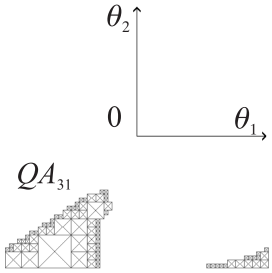

Fig. 15: Free Q-aspect QA_{31}

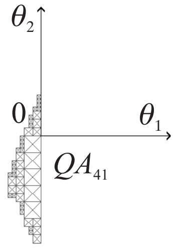

Fig. 16: Free Q-aspect QA_{41}

In a free W-aspect, the manipulator can achieve any continuous trajectory since the free W-aspects are free of collision and singularity by definition. For instance, the trajectory

(CD) is feasible (Figure 10). On the other hand, if a trajectory is given which does not lie in a same aspect like the trajectory (EFG) (Figures 13 and 14), a change of working mode is required. The change of working mode makes the output link leave the trajectory. Thus, the continuous trajectory (EFG) is infeasible.

5 Conclusion

In this paper, new notions were defined to take into account the singularities and the presence of internal/external collisions in the moveability analysis of fully parallel manipulators. To take into account collisions, the collision free space, the free workspace and the free joint space were defined. Then, by using the notion of working modes, a general definition of free aspects was introduced. The free aspects are defined as the maximal sets of $W \cdot Q$ which are free of singularities and collisions. Upon projecting these free aspects onto the workspace (resp. the joint space), we get the W- aspects (resp. the Q-aspects). The W-aspects were shown to be the regions of the workspace were any continuous trajectory is feasible. These sets are of high interest for the trajectory planning, the control and the design of fully-parallel manipulators. Further research work is conducted by the authors on the design of fully-parallel manipulators to minimize the effects of internal collisions.

References

[1] Merlet J-P. "Les robots parallèles" HERMES, seconde édition, Paris, 1997.

[2] Gosselin, C. and Angeles, J. "Singularity analysis of closed-loop kinematic chains" IEEE Transactions On Robotics And Automation, Vol. 6, No. 3, June 1990.

[3] Wenger, Ph. and Chablat, D. "Uniqueness Domains in the Workspace of Parallel Manipulators" IFAC-SYROCO, Vol. 2, pp 431-436, 3-5 Sept., 1997, Nantes.

[4] Angeles, J. "Fundamentals of Robotic Mechanical Systems" SPRINGER 97.

[5] Borrel, P. "A study of manipulator inverse kinematic solutions with application to trajectory planning and workspace determination" Proceeding IEEE International Conference on Robotic And Automation, pp. 1180-1185, 1986.

[6] D. Chablat, Ph. Wenger "Working modes and aspects in fully parallel manipulators" to appear in Proceeding IEEE International Conference of Robotic and Automation, Mai 1998.

[7] Chablat, D. and Wenger, Ph. "Domaines d'unicité des manipulateurs parallèles" IRCyN Internal Report, $N°96.13$, Nantes, 1996.